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Cellular Space Models of Self-Replicating Systems

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Abstract. Biological organisms are the most familiar examples of self-replicating systems, and until the late 1940s, the only instances formally researched. At that time, mathematicians and scientists began studying artificial self-replicating systems when it became desirable to gain a deeper understanding of how complex systems are able to form and evolve. Initial models consisted of abstract logical machines, or automata, embedded in cellular spaces. The large complexities seen in these early models agreed with the intuition that self-replication was an inherently complex process. Later, it was learned that much of the complexity was due to the imposition of artificial requirements. This paper traces developments from complex, early models of self-replicating systems in cellular spaces to recent, less complex models. As a survey of past models, this paper provides an overview of numerous self-replicating systems as well as some recent models that rely on emergent processes and artificial evolution.

1. Introduction

The brilliant mathematician John von Neumann initiated the formal study of artificial self-replicating systems in 1948, and before his untimely death in 1957, he had produced the first logical design of a self-replicating automaton [**31**]. Over the decades since this demonstration, theoretical and modeling studies have led to progressively simpler and smaller structures [**7, 3, 13, 23**]. They have produced structures that do problem solving while replicating [**6, 20, 27**], structures that were produced automatically via artificial evolution [**10, 16**], as well as demonstrated that self-replicating structures can emerge from a "sea" of non-replicating components [**5**]. The focus of this paper is to describe these and other models of artificial self-replication, limiting our attention to those embedded in cellular spaces. First we look at look at why these models are worthy of investigation and the historical trend away from complexity.

A better understanding of self-replicating systems could be useful in a number of ways, for both theoretical and practical purposes. Von Neumann was interested in understanding the build-up and evolution of extremely complex systems. Since biological organisms were known to be of enormous complexity and had complicated self-replication processes, he thought it natural to research self-replicating

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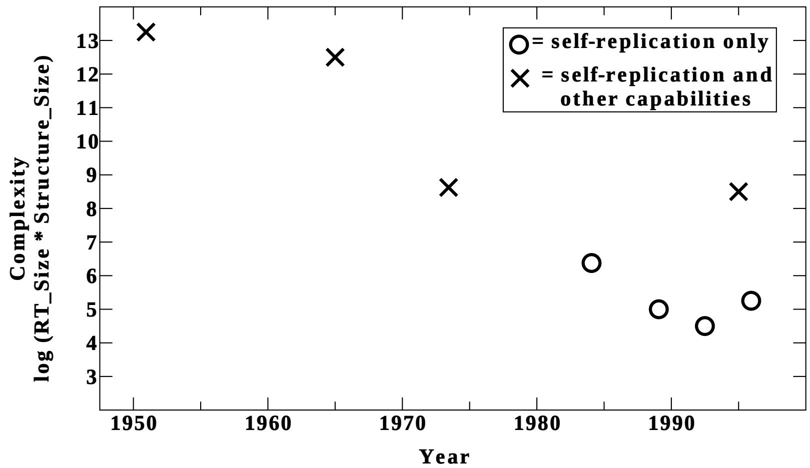

FIGURE 1. Plot of self-replicating system complexity for cellular automata models. From left to right, the × symbols are von Neumann [**31**], Codd [**7**], Vitányi [**29**], Perrier *et al.* [**20**], and the O symbols are Langton [**13**], Byl [**3**], Reggia *et al.* [**23**], Lohn and Reggia [**17**].

systems. He was especially interested in how a complex system could be constructed out of numerous simple parts. More recently, other reasons for studying abstract models self-replication were posited. The field of artificial life was largely borne out of studies of such models. Subsequent researchers in artificial life found self-replicating structures to be a natural goal, especially for studying bottom-up, synthetic biologies. On a more practical bent, research on self-replicating structures could be useful in areas such as molecular-scale manufacturing [**9**], programming massively parallel computers [**22**], and computer virus research [**12**]. Researchers in molecular-scale manufacturing (also called nanotechnology), have discussed the potential of self-replicating systems: "If assemblers are to process large quantities of material atom-by-atom, many will be needed; this makes pursuit of self-replicating systems a natural goal." [**9**]. Having self-replicating computer programs that can be acted upon by digital evolution [**22**] could allow easier programming of massively parallel computers. Evolutionary bred self-replicating programs would breed on the parallel computer and the programs that most satisfy a set of requirements would be allowed to survive and replicate. Researchers have also investigated self-replicating structures to aid in understanding biomolecular mechanisms of reproduction and the origins of life [**11**].

Cellular space models of self-replicating systems have progressed from complex models to less-complex models. This trend is apparent in Figure 1, where complexity is plotted against time for cellular automata models. There are certainly other measures of complexity one could chose, but we have defined it to be the product of rule table size and structure size, plotted logarithmically. As can be seen, models designed only for self-replication are lowest in complexity, with the least complex of the others having three orders of magnitude more complexity.

The remainder of the paper is divided into three main sections. In section 2 we present background material regarding cellular space models and self-replicating

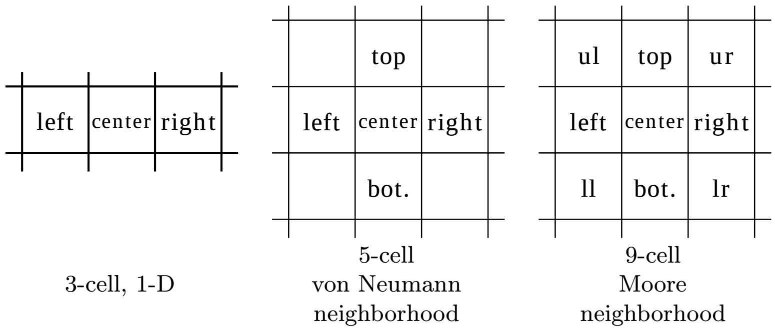

FIGURE 2. Common neighborhood patterns in 2-D cellular space models.

structures. In section 3 we present a series of brief case studies beginning with the work of von Neumann and continuing through till present day. Included here are models that are purely self-replicators, and those that provide additional functionality in addition to self-replication. In section 4 we summarize the paper and discuss potential research directions in the field.

2. Background

2.1. Cellular Space Models. A cellular space is a tessellation of cells containing finite state automata that interact with each other. The key properties of cellular space models are: strictly local interactions (resulting in emergent behavior), rule based automata (usually deterministic), high parallelism, simple automata (in general), and discretized space and time. Cellular automata (CA) models are the most widely studied models and constitute the majority of cellular space models. There are also numerous variations of the standard cellular automata model – for example: models that embed complex automata (e.g., a CPU with registers) within the cellular space, and models that allow other events or operators to act in addition to state transitions. These models are still cellular space models, yet it would not be proper to call them cellular automata.

2.2. Cellular Automata. Von Neumann co-invented cellular automata with Stanislaw Ulam as a medium in which to investigate and design complex systems such as self-replicating machines. Cellular automata are a class of spatially-distributed dynamical system models in which many simple components interact to produce potentially complex patterns of behavior [**7, 32**]. In a cellular automata model, time is discrete, and space is divided into an N-dimensional lattice of cells, each cell representing a finite state machine or automaton. All cells change state simultaneously with each using the same function δ or rule table to determine its next state as a function of its current state and the state of neighboring cells. This set of adjacent cells is called a *neighborhood*, the size of which, n, is commonly five or nine cells in 2-D models (see Figure 2). By convention, the center cell is included in its own neighborhood. Each cell can be in one of k possible states, one of which is designated the quiescent or inactive state. When a quiescent cell has an entirely quiescent neighborhood, a widely accepted convention is that it will remain quiescent at the next time step.

CTRBL	C′	CTRBL	C′	CTRBL	C′	CTRBL	C′
00000	0	01000	1	10000	1	11000	0
00001	1	01001	0	10001	0	11001	1
00010	1	01010	0	10010	0	11010	1
00011	0	01011	1	10011	1	11011	0
00100	1	01100	0	10100	0	11100	1
00101	0	01101	1	10101	1	11101	0
00110	0	01110	1	10110	1	11110	0
00111	1	01111	0	10111	0	11111	1

TABLE 1. Example CA rule table for the parity function.

The CA rule table is a list of transition rules that specify the next state for every possible neighborhood combination. In a 2-D, 5-neighbor model the individual transition rules would be of the form CTRBL $\rightarrow$ C′, where CTRBL specifies the states of the Center, Top, Right, Bottom, and Left positions of the neighborhood's present state, and C′ represents the next state of the center cell.

The underlying space of CA models is typically defined as being isotropic, meaning that the absolute directions of north, south, east, and west are indistinguishable. However, the rotational symmetry of cell states is frequently varied. Strong rotational symmetry implies that all cell states are unoriented, meaning that each neighbor to a cell has no distinguishable position. Weak rotational symmetry implies that at least one cell state[1] is directionally oriented, meaning that the cell designates specific neighbors as being its top, right, bottom, and left neighbors. For example, the cell state designated $\uparrow$ in von Neumann's work is weakly-symmetric and thus permutes to different cell states $\rightarrow$, $\downarrow$, and $\leftarrow$ under successive 90° rotations. It represents one oriented *component* that can exist in four orientations. In CAs that contain both weak and strong rotationally symmetric states, it is common to represent the "strong" states using symbols that appear rotationally symmetric (e.g., $\circ$, $+$, $\times$), and the "weak" states (components) using symbols that are not rotationally symmetric (e.g., $\uparrow$, `A`, `L`).

As an example CA, consider the parity function where a cell's next state is one if there are an odd number of ones in the five cell neighborhood. The rule table for this function is shown in Table 1, and the configurations at four points in time are shown in Figure 3. It is interesting to note the complex patterns that arise from a simple two state, five neighbor function. Such dynamics illustrate the emergent behavior that is typical of many cellular automata simulations. Also note that CAs are typically very sensitive to initial conditions. For example, even a slight change to the t=0 state of the parity example will drastically change the dynamics.

2.3. Self-replicating structures. There is no universally accepted definition of a self-replicating structure, but we can qualitatively describe a generic self-replicating structure as follows. The structure itself is typically represented as a configuration of contiguous non-quiescent cells (see the five-component structure in Figure 4). As the space iterates, the structures goes through a sequence of steps to

[1]The quiescent state is always a strongly rotation symmetric cell state and is generally included in CA models with weak rotational symmetry.

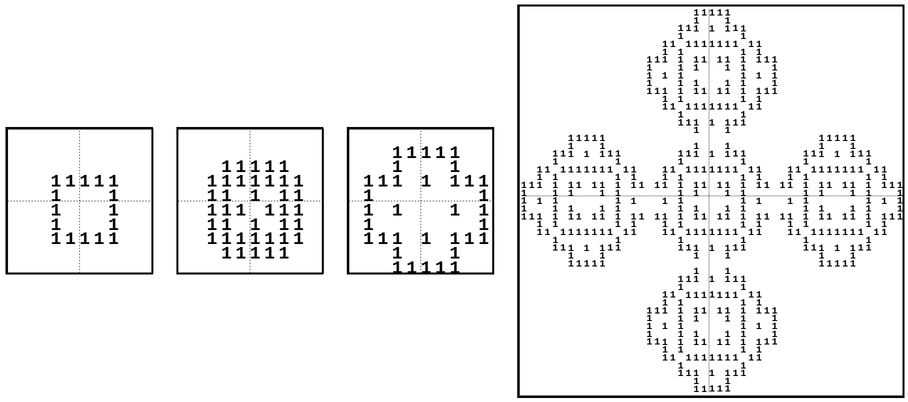

FIGURE 3. Configurations at t=0, t=1, t=2, and t=22 for the parity function.

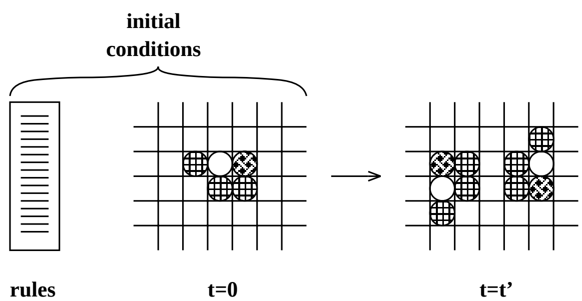

FIGURE 4. Illustration of self-replicating structure.

form a replicant. At some time t', copy of the original structure appears isolated, and possibly rotated.

The issue of triviality was circumvented in early models by requiring universal computation and universal construction. Inspired by biological cells, more recent models (those starting with [**13**]) have abandoned this requirement by insisting that an identifiable instruction sequence be treated in a dual fashion: interpreted as instructions (translation), and copied as raw data (transcription). As with unsheathed loops (i.e. loops lacking an outer covering of non-quiescent cells), one can also consider the instruction sequence and the structure itself to be the same, and thus the structure's components directly influence its self-replication process.

2.4. Chronological Summary. A summary of some previous research involving self-replicating structures in cellular space models is shown in Table 2. Most models shown have been 2-D CAs with strong rotational symmetry. In models with weak rotational symmetry, each rotated cell state is counted in the "states per cell" column. The sizes of the self-replicating structures are measured in non-quiescent cells, and are sometimes estimates since some systems were never implemented. The listed models are primarily designs and implementations, though existence proofs of self-replicating structures have appeared (e.g., [**25**]). The models shown include

Year	Model Type	Rot. Sym-metry	States per Cell	Neigh-borhood size(s)	Struc-ture size(s)	Func-tion-ality*	Refs.
1951	CA	weak	29	5	$> 10^4$	a	[**31, 21**]
1965	CA	strong	8	5	$> 10^4$	a	[**7**]
1966	CT-mach.	weak	$\approx 10^{100}$	5	$\approx 10^2$	a	[**1**]
1973	CA	strong	8	5	$> 10^4$	s	[**29**]
1976	α-Univ.	strong	5	var.	≈ 60	s	[**10**]
1984	CA	strong	8	5	86	s	[**13**]
1989	CA	strong	6	5	12	s	[**3**]
1993	CA	†	6,8	5,9	5–48	s	[**23**]
1995	CA	strong	10	9	52	a	[**27**]
1995	EA	weak	9,13	5	2,3	s	[**16**]
1995	non-uni. CA	strong	3	9	5	s	[**24**]
1996	CA/W-mach.	strong	63	5	127	a	[**20**]
1997	CA	weak	192	9	4,8,...	a	[**5**]
1997	CA	weak	12	5	4,8	a	[**19**]

* s=self-replication, a=capabilities in addition to self-replication.
† Both strong and weak rotational symmetries were investigated.

Table 2. Comparison of some self-replicating structures in cellular space models.

variations of cellular automata, and will be discussed in the next section. Briefly, CT-machines are programmable finite automata with registers, α-Universes are CAs augmented with chemistry-like operators, non-uniform CAs allow cells to have differing rules, and W-machines are Turing machine models that are programmable using high-level instructions.

3. Review of Models

The section presents a series of brief reviews of self-replicating structures in cellular space models. Beginning with the pioneering work of von Neumann and continuing to recent models, the trend toward less complex structures is evident. Diagrams showing the space-time iteration of the cellular space are shown for most of the models surveyed here.

3.1. Von Neumann's Model. Among his other interests in the late 1940s, John von Neumann wanted to gain a deeper understanding into the nature of complex systems. He was keenly interested in how such such systems formed and evolved from collections of numerous simple components [**30, 31**]. This interest led him to investigate machines that could construct other machines, the so-called universal constructors. Within the set of universal constructors are a special subset of machines – the self-replicating machines. Partly from his interest in biological organisms, he devoted much time and energy to the study of self-replicating machines. His seminal work in this area formed the cornerstone of what is known today as artificial life – synthesis-based approaches to theoretical biologies.

The logical design of his self-replicating automaton consisted of a 29-state, 5-neighbor, weakly rotation symmetric CA, consisting of many millions of cells. An overview of this machine is seen in Figure 5, where the four main areas of the

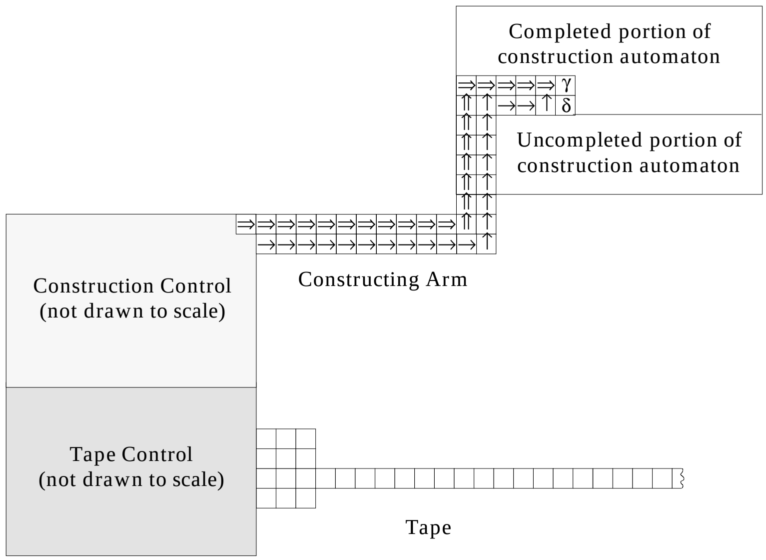

FIGURE 5. Overview of von Neumann's self-replicating automaton (adapted from [**2**]).

machine are identified: tape, tape control section, construction control section, and constructing arm. The tape contained the description of the desired machine to construct. The tape control area read and interpreted the tape as well as transferred excitation signals. The construction control area extended and sent signals to the construction arm.

After supplying a correctly-programmed tape to the machine, the sequence of steps needed to have the machine self-replicate were as follows: *i*) reading and interpreting the input tape, *ii*) constructing new cells in the quiescent area, *iii*) "rewinding" the tape, then copying it, *iv*) attaching tape copy to newly constructed portion, *v*) signaling the newly constructed portion that construction had completed, and *vi*) retracting the construction arm.

3.2. Codd's Model. E. F. Codd introduced a simpler universal constructor embedded in an 8-state, 5-neighbor, 2-D strongly rotation symmetric cellular automata, consisting of 100,000,000 cells [**7**]. A simplification to approximately 95,000 cells appeared later [**8**]. It shared behavioral similarities to von Neumann's model, but with reduced complexity. The design was influenced by neurophysiology of animals, and one of the notable features was the inclusion of sheathed signal paths.

3.3. Vitányi's Model. The model of Vitányi [**29**] was an example of a sexually-reproducing cellular automaton. This model employed an 8-state, 5-neighbor cellular space and requires tens of thousands of cells for the two structures. It was argued that in transitioning from asexual to sexual reproduction, a change was needed in the number and structure of instruction tapes. The model specified **M**-type (male) and **F**-type (female) automata, each containing two, nearly identical instruction

```
 ########
#O+ OL OL#
# ###### #
#+#    #O#
#O#    #O#
# #    #O#
#+#    #O#
#O######O#####
# +O +O +OOOOO#
 #############
```

FIGURE 6. Initial configuration of Langton's self-replicating loop. An identifiable instruction sequence `++++++LL` is readily seen embedded in the core of `O` states within the sheath.

tapes. Although the automata were quite complex, the model showed that sexual reproduction of automata is possible, and that the recombination process was somewhat similar to that of nature.

3.4. Langton's Self-replicating Loop. The relatively recent resurgence in modeling self-replicating structures is mainly due to studies conducted by Christopher Langton. By recognizing that computation universality was not required to obviate triviality, he was able to devise a vastly simpler self-replicating structure in cellular automata [**13**]. Using concepts from Codd's work, he derived an 8-state, 86-cell sheathed loop that requires 108 replication rules, orders of magnitude simpler than previous models. The initial state of the loop is depicted in Figure 6, and the first six time steps are shown in Figure 7. Figure 8 shows time step 151 where the first replicant has appeared.

3.5. Byl's Model. Byl made further refinements and derived a six state, twelve cell self-replicating structure that required 57 replication rules and had a single sheath [**3**]. Figure 9 shows the initial configuration of the loop. Figure 10 shows the first 24 time steps of the loop, and Figure 11 shows the first replicant produced at time step 25.

3.6. Reggia's Self-replicating Loops. Further simplification of self-replicating loops was found by deriving unsheathed loops, and varying symmetry conditions [**23**]. This study verified that both strong and weak rotational symmetries can yield simple self-replicating structures. The smallest structures found were a 6-state, 5-neighbor, 5-cell unsheathed loop under strong rotational symmetry, and an 8-state, 5-neighbor, 6-cell unsheathed loop under weak rotational symmetry. Figure 12 shows the first ten time steps for a structure with a six component unsheathed loop embedded in an 8-state, 5-neighbor CA space with weak rotational symmetry. Figure 13 shows the colony that forms at time step 84.

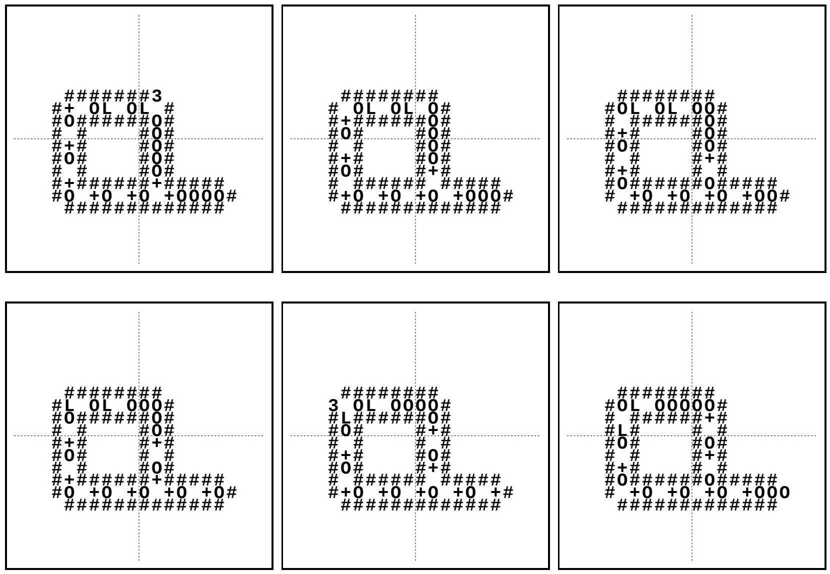

FIGURE 7. Time-steps 1 through 6 for Langton's self-replicating loop. The instruction sequence circulates counterclockwise on successive steps.

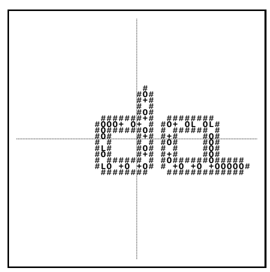

FIGURE 8. Time-step 151 shows the first replicant.

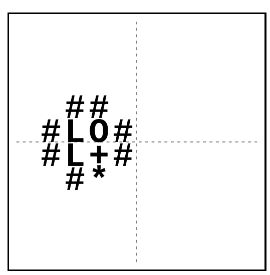

FIGURE 9. Initial configuration of Byl's self-replicating loop.

3.7. Tempesti's Model. In [**27**] a 6-state, 9-neighbor, 52-cell self-replicating structure is reported that is augmented with additional construction and computational capabilities. It is similar to Langton's self-replicating loop, except in the following ways. First, it has the ability to execute programs in offspring structures.

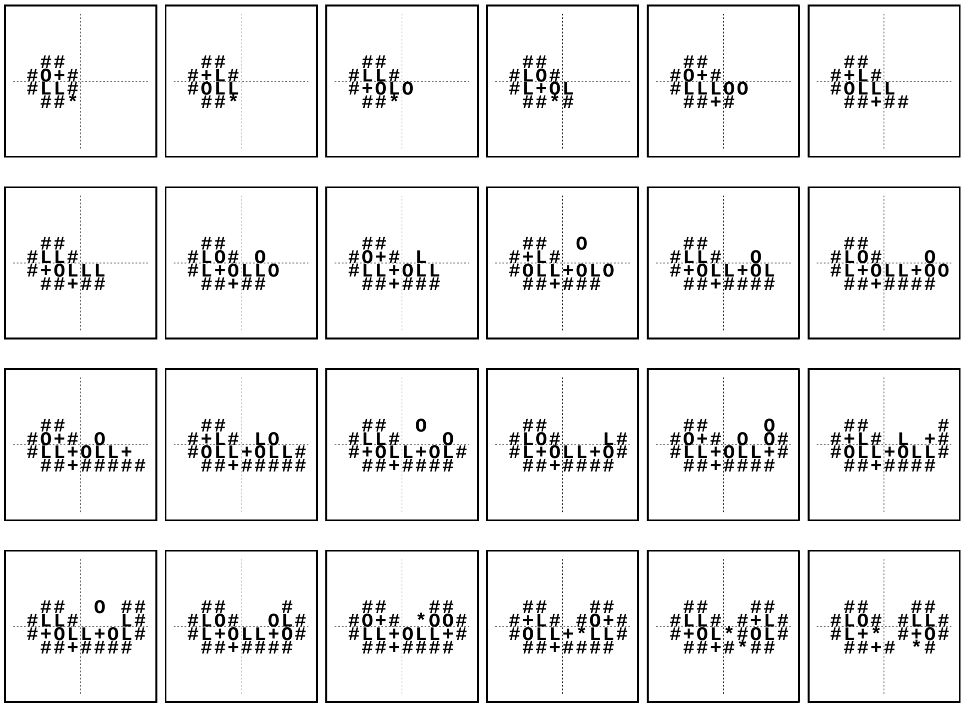

FIGURE 10. Time steps 1 through 24 of Byl's self-replicating loop.

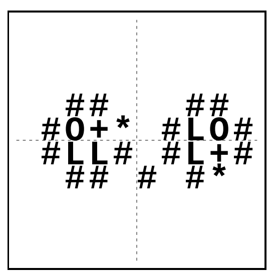

FIGURE 11. First replicant produced after 25 time steps.

Second, it uses a single interior sheath (instead of a double sheath), which is constructed prior to the signal being sent out. Third, instead of parent loops becoming quiescent, they remain active and are capable of program execution. Fourth, the construction arm extends in four directions simultaneously, as opposed to a single direction. Figure 14 illustrates the structure of the loops at two points in time.

3.8. Arbib's CT-machine. Arbib [**1**] noticed that the large degree of complexity of von Neumann's and Codd's self-replicating automata could be greatly reduced if the fundamental components were more complex. He developed a model in which automata are analogous to biological cells, as opposed to molecules. Thus, his automata are very complex, and his description and proofs regarding self-replicating functionality are much shorter than von Neumann's and Codd's.

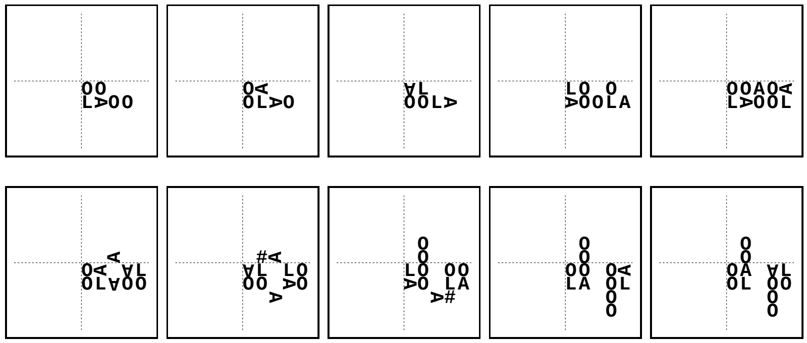

FIGURE 12. Time-steps 0 through 9 for an unsheathed loop structure [**23**]. The number of replication rules for this structure is 58.

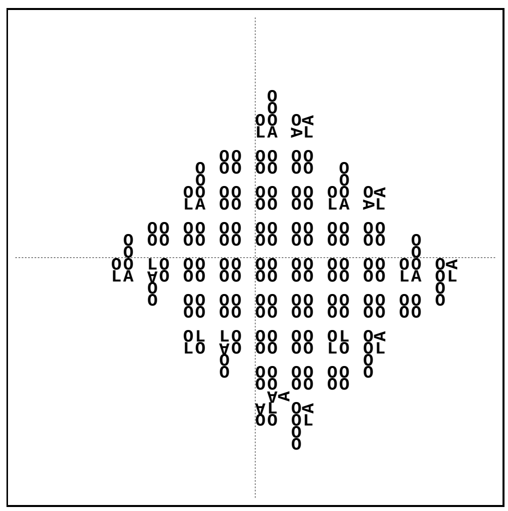

FIGURE 13. The colony that forms later (time step 84) in the development of the unsheathed loop structure from Figure 12.

His automata had approximately 10^{100} states and were capable of both universal computation and construction. The automata are embedded in a 2-D cellular space model called Constructing Turing machines, or CT-machines [**28**]. Each cell in this space (Figure 15) contains a finite-state automata that execute short 22-instruction programs (Figure 16). Instructions consist of actions such as weld and move, and internal control constructs such as if/then and goto. Self-replication occurs when individual CT-machines copy their instructions into empty cells. Composite structures consisting of multiple CT-machines are able to move as one unit since individual automata can be welded to each other. Each of the four components is constructed out of identical automata, each programmed specifically for the appropriate function `W` denotes weld positions, `BR` denotes bit register, module is programmed using instructions such as weld, emit, move, goto. The machine operates as follows. Since cells can be "welded", a tape can be formed. The control

```
                                               1
                                               1
     1                       1                 1
  dddd2ddd              1dd2ddddd        dddd2ddd
 1d111111d               d111111d       1d111111d
  d1    1d               d1    12        d1    12
  21    1d               d1    1d        d1    1d
  d1    12               d1    1d        d1    1d
  d1    1d               21    1d        21    1d
  d111111d1              d111111d        d111111d111
  ddd2dddd               ddddd2dd1       ddddd3dd
   1                     1               1


   time = 0                     time = 121
```

FIGURE 14. Time steps 0 and 121 of Tempesti's loop. State `d` represents the data state.

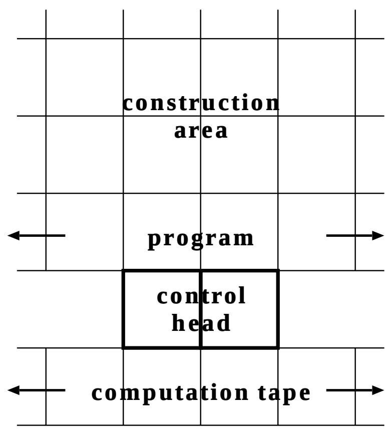

FIGURE 15. Overview of embedded automata in the CT-Machine model.

head can read and write the computation tape in the same manner as a Turing machine. The construction area is initially quiescent. Program cells can only write into the construction area, and these write operations are equivalent to the placing of new components.

3.9. Holland's Model. In the mid 1970s, John Holland developed a theoretical framework for the spontaneous emergence of a class of artificial self-replicating systems [**10**]. Holland defines a set of model "universes" containing abstract counterparts to rudimentary chemical and kinetic mechanisms such as bonding and movement. He wanted to loosely model natural chemical processes (diffusion, activation) acting on structures composed of elements (nucleotides, amino acids) to show that even with random agitations, the tendency of such a system would not be sustained randomness, but rather, life "in the sense of self-replicating systems undergoing heritable adaptations."

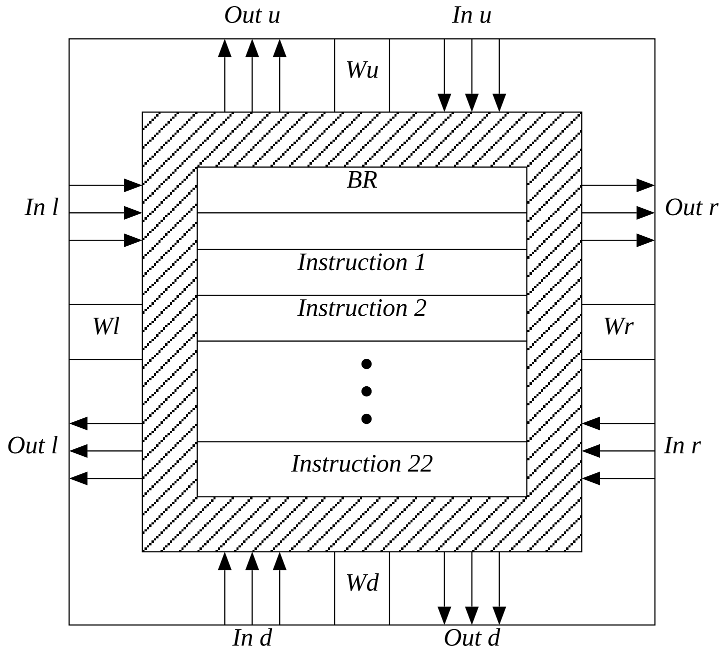

FIGURE 16. Automaton in Arbib's CT-Machine model.

...	-	0	:	1	0	0	1	-	N_0	N_1	-	-	0	:	...

Element	Codon
0	N_0N_0
1	N_0N_1
:	N_1N_0

FIGURE 17. An example of a few cells from an α-Universe

The α-Universe cellular space model represents cell states as elements that are logical abstractions of physical entities (e.g. atoms) and obey the conservation of mass. Figure 17 shows an example of part of an α-Universe. Elements are the fundamental units with codons encoding the elements as seen in the illustration.

Interactions among the elements are strictly local as in CA, but some are localized to aggregate structures (strings of bonded elements). Elements behave as automata during the first of three "phases" of each discrete time-step. During the second and third phases, they are acted upon by the four operators: bonding, movement, copy, and decode. As an example, the "copy" operator becomes activated if the sequence `-0:`$e_1e_2\cdots e_l$`-` forms (e_i being one of the three elements), and it would cause elements to be reshuffled so that a codon-encoded copy of the string $e_1e_2\cdots e_l$ would be assembled.

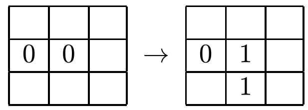

FIGURE 18. Example transition rule.

```
                                                  1
11     11      11       11       11     11       11
111    1110    11100    11101    11101  11011    11 11
                            1       11     11       11
                                           1        1
                                                    0

t=0     t=1     t=2      t=3      t=4    t=5       t=6
```

FIGURE 19. Self-replicating structure comprised of 5-cells that uses 10 transition rules in a 2-state, 9-neighbor non-uniform CA model.

Holland parameterizes important aspects of the α-Universes and then uses these to derive formulas that predict the expected time required for emergence of a self-replicating system. Substituting reasonable values into his derivations, a waiting time of 1.4×10^{43} time-steps is computed (no emergence). Relaxing the requirement from *fully* self-replicating to *partially* self-replicating, a waiting period of 4.4×10^8 time-steps (4.4×10^8 seconds is about 14 years) is obtained. Since this is a reasonable amount, it lends credence to spontaneous emergence of self-replicating structures in general, given that Holland's model and derivations are accurate. In [**18**], an empirical investigation claims that some of the conjectures were flawed. Regardless of whether the original analysis is valid, it remains one of the only studies of its kind reported to date and raises important theoretical questions regarding emergence of self-replicating structures.

3.10. Sipper's Model. Sipper describes a self-replicating loop motivated by Langton's work [**24**]. The cellular space model is a modified cellular automata whereby: the space iterates in discrete time with cells updated in a local, synchronous manner, but unlike a CA, a given cell can change a neighboring cell's state (Figure 18 shows an example transition rule). Also, a cell can copy its rule into a neighboring cell (non-uniform CA). Figure 19 shows the five component structure in its self-replication process.

3.11. Perrier's Self-replicating Loop. Perrier reports a 63-state, 5-neighbor, 127-cell self-replicating structure, exhibiting universal computation[**20**] (see Figure 20). Universal computation is achieved by using Turing machine model called the W-Machine which is programmed using a small instruction set. Complexity is reduced by eliminating requirement of construction universality. The loop structure self-replicates in the same manner as Langton's self-replicating loop. Program and data tapes are copied using transmitted signals. After a daughter structure is produced, it can execute a W-Machine program.

```
 ........
.70170170.
.1......1.
.1.    .7.
.1.    .0.
.1.    .1.
.1.    .7.
.0......0....
.410410710711.
.A...........
.P.    .D.
.P.    .D.
.P.    .D.
.P.    .D.
.P.     .
.P.
.P.
 .
```

FIGURE 20. Perrier's self-replicating loop. The structure consists of three parts: loop, program, and data: `D` represents a data cell, `P` represents a program cell, and `A` represents the position of the program.

3.12. Emergent Self-Replicating Structures. Previous self-replicating structures have always been initialized with a pre-defined structure. Chou and Reggia [**5**] investigates whether there exists a CA transition rule that can promote the emergence of self-replicating structures from a *randomly* initialized CA space. Using a new cellular automata programming language and development environment called Trend, CA transition rules were found that: *i*) support replication of different-sized structures, *ii*) show growth of small structures into larger ones, *iii*) allow interactions between structures, and *iv*) are robust: independent of space size and initial component density. Figure 21 shows some of the emergent self-replicating loops that emerged.

3.13. Self-Replicating Loops: Problem Solving and Artificial Selection. Recent work on models that incorporate problem-solving capabilities into self-replicating loops has yielded loops that can solve satisfiability (SAT) problems. Previous models incorporated a fixed "program" that is copied unchanged to replicants. Chou and Reggia [**6**] demonstrate solutions to the SAT problem in which replicants receive partial solutions that are modified during replication, and artificial selection: promising solutions proliferate, failed solutions are lost. The environment selects satisfied clauses by using "monitor" cells which destroy unsatisfied loop fragments. Figure 22 depicts how the SAT problem predicate $Q = (\neg x_1 \vee x_3) \wedge (x_1 \vee \neg x_2) \wedge (x_2 \vee \neg x_3) \wedge (x_4 \vee x_4) \wedge (\neg x_4 \vee \neg x_5) \wedge (x_5 \vee \neg x_6)$ is solved using 4 by 4 self-replicating loops.

3.14. Automatic Discovery of Self-replicating Structures. The question of automatically discovering self-replicating structures is examined in [**16, 17**].

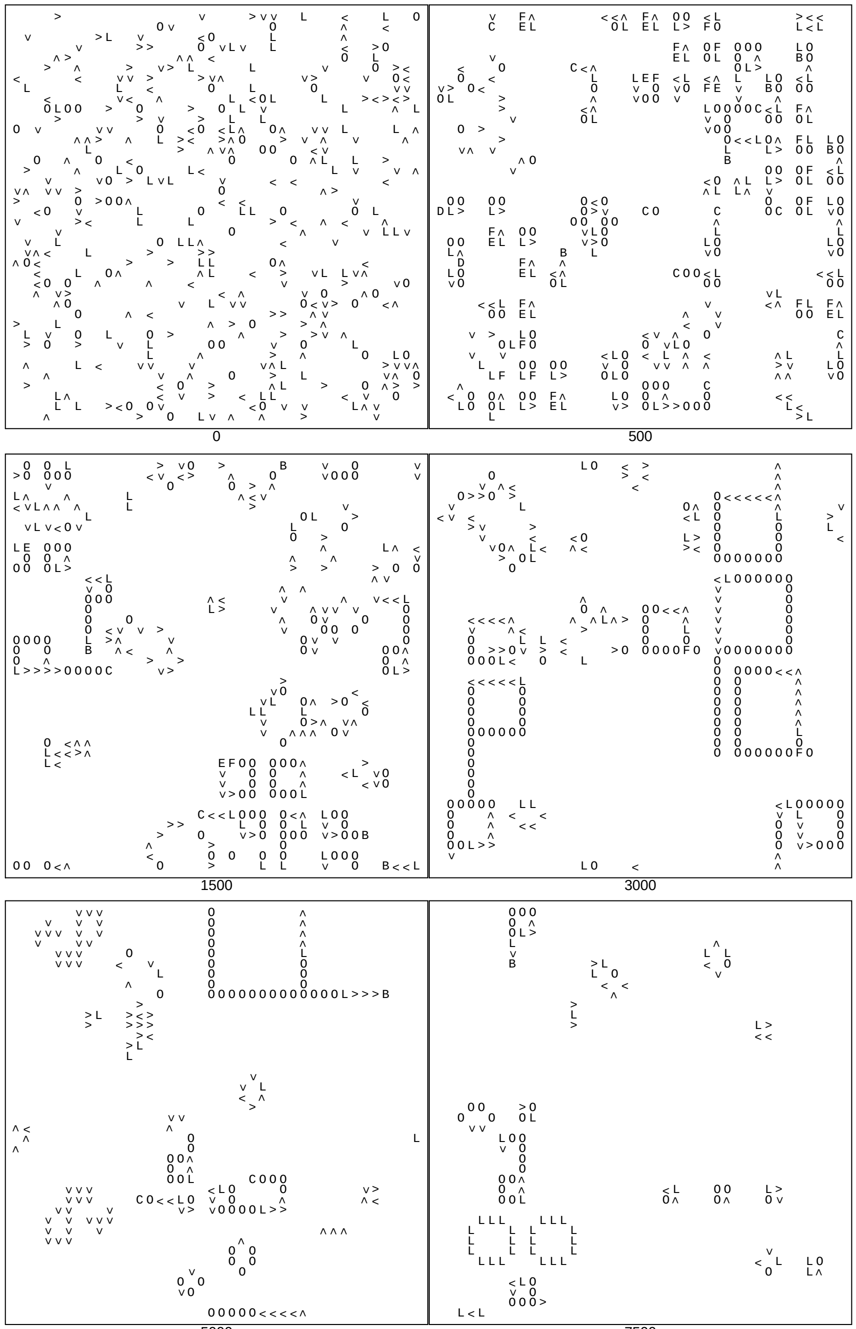

FIGURE 21. Emergent self-replicating loops: at t=500 2×2 and 3×3 can be seen, at t=1500 4×4 loops, at t=3000 8×8 loops, and st t=5000 a single 10×10 loop can be seen. At t =7500 large loops have been replaced by smaller ones. Illustration from [**5**].

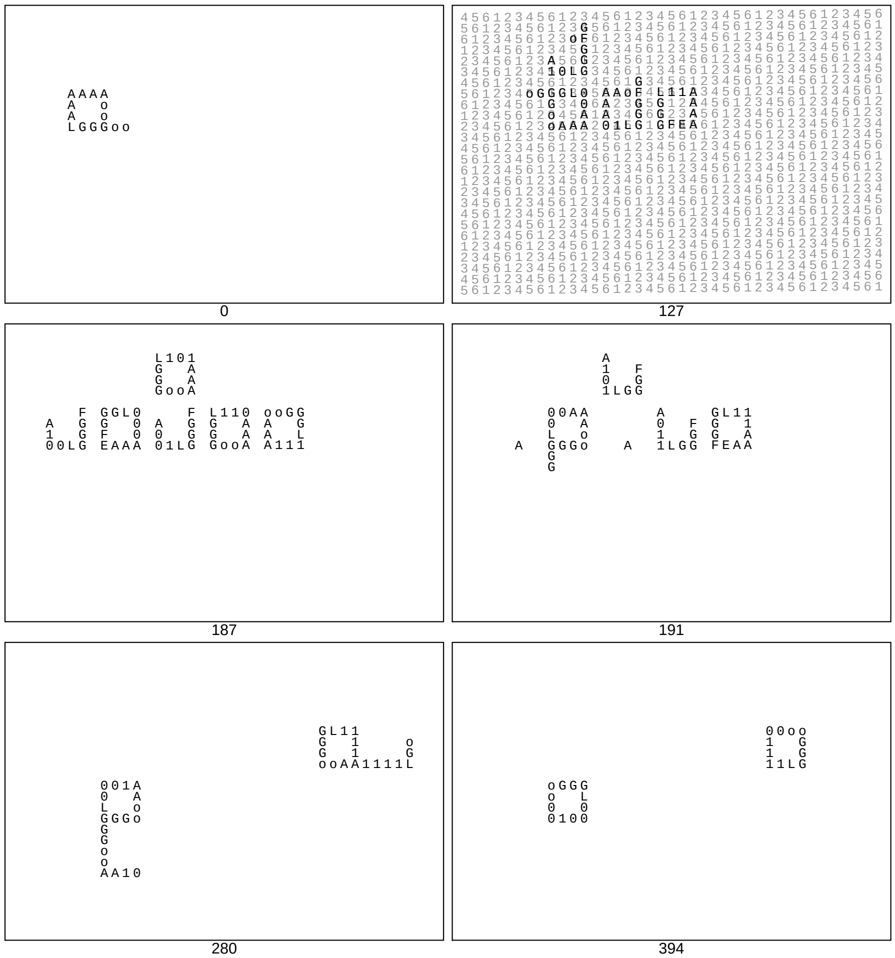

FIGURE 22. Self-replicating loops during SAT problem solving. The initial 4 by 4 loop contains explored bits AAAAAA. The frame at t=127 shows the population of "monitor" cells which function to remove unsatisfied clause bits. New loops are generated and tested by the monitor cells (not shown in last four frames). The two solutions, 000100 and 111100 are found at t=394. Illustration from [**6**].

In all such past models, the underlying transition rules have been manually designed, a process that is very difficult and time-consuming, and is prone to subjective biases of the implementor. This research introduced the use of genetic algorithms to discover automata rules that govern emergent self-replicating processes. Identification of effective performance measures (fitness functions) for self-replicating structures was a key challenge in this problem. A genetic algorithm using multiobjective fitness criteria was applied to automate rule discovery. The results show that novel self-replication processes were uncovered by the genetic algorithm. For example, some of our structures both rotate and move during self-replication, and some leave around unused components (debris) which promote the formation of new structures. Such behaviors, which have not been used or considered in past manually-designed self-replicating structures, are especially interesting, suggesting that evolutionary computation can discover novel design concepts of general value. Figure 23 shows an examples of an automatically discovered structure.

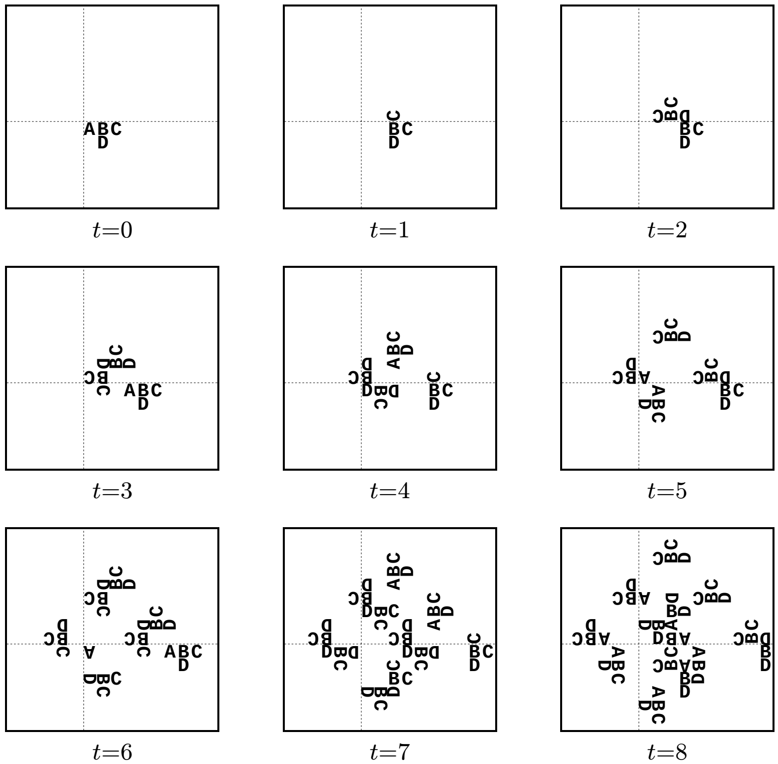

FIGURE 23. Four-component self-replicating structure. The seed structure moves towards the right on successive time steps and produces two replicants: the first is seen at t=4 and then again at t=7 along with the second replicant (upper right quadrant of each respective frame). These replicants are rotated 90° counter-clockwise and proceed upward. During the production of the first replicant, debris forms (near coordinate system origin of t=3 and t=4) and coalesces into two structures seen at t=5, lower left. One structure moves downward and attempts to self-replicate but due to crowding, is unable. The other moves to the left and produces its first replicant at t=8 (lower left quadrant).

4. Discussion

Many of the models of self-replicating structures appearing in the literature have been described in the preceding sections. From the early models to the present day, the progressive simplification of self-replicating structures in cellular automata is apparent. This was accomplished first by relaxing the requirements of construction and computation universality, and later by reduction of structure size. We've seen how self-replicating structures have been constructed in cellular space models other than cellular automata, and how automatic discovery methods have been employed with respect to searching for self-replicating structures.

There are many directions for further research in this area. Investigating minimal structure size in cellular automata structures, the effect of varied neighborhoods (size and shape) and varied seed structures are logical extensions of some of the previous work. Larger questions regarding the choice of cellular space models, for example, stochastic automata, would appear worthwhile. In the realm of automatic discovery, investigation of other search techniques would be of interest since the fitness landscapes in these problems are poorly understood. Another area is biochemical simulation: a few promising studies have appeared in which modified cellular automata models are used to mimic biochemical interactions and simulate template-directed oligonucleotide replication.

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